ISSN 2229-5518

Revealing Facts of Hydrodynamic Pressure: Law of Isotropic Pressure

Madiha Gohar *,Iqra Shahzadi, Usfar Bilal[2], Zill-e-Huma [3].

* Directorate of Distance Education, Department of Mathematics, Sub Campus Sahiwal-57000, Bahauddin Zakariya University, Multan, Punjab, Pakistan.
1 Department of Earth and Environmental Sciences, University of the Punjab, Lahore-54590, Punjab, Pakistan.
2 Department of Mechanical Engineering, COMSATS Institute of Information and Technology, Sahiwal-57000, Punjab, Pakistan.
3 Department of Electrical Engineering, Information Technology University of the Punjab, Lahore-54590, Punjab, Pakistan.

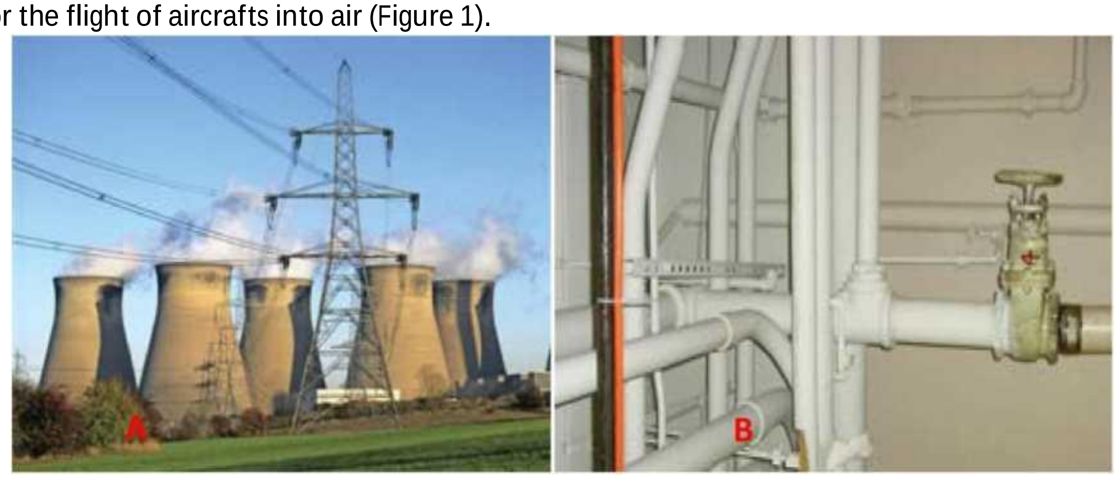
Corresponding Author: Madiha Gohar
Email: madihaaohar80@gmail.com
Telephone: +92 331 704 3244

Abstract: The channel flow, pipeline flow, flow of water in pumps to reach the water to the water tank on the top of the roof, ship hull design, oceanography, hydropower system and many other systems and branches of fluid mechanics which deals with the study of moving fluids are working on the base of the law of hydrodynamic pressure. The law could be stated by the three main equations i-e $p=p_x$, $p=p_y$ and $p=p_z$. Taking pressures p_x,p_y,p_z in YZ,ZX and XY planes after that applying Newton's second law of motion and using the volume of Tetrahedron we can easily conclude the pressure acting along the force F_x,F_y and F_z. The manuscript describes the effect of these forces in presence of pressure on moving fluids.

--------------------- ♦ -----------------------

Introduction:

Hydrodynamics is a branch of fluid mechanics in which the flow of incompressible fluids is discussed in motion. "The pressure due to the random motion of molecules, these molecules exerts a pressure on fluid which is called Hydrodynamic pressure". The foundational axioms ofdynamic pressure are based onconservation law of mass, conservation of linear momentum(Newton's second law of motion), and conservation law of energy. Leidenfrost has a great contribution to understand the hydrodynamic pressure. After Leidenfrost, Kistemaker studied the phenomena [5] and he edit the experiments and research of Leidenfrost.The marine engineers and architectures work on the hydrodynamic pressure and have designed the bodies of ships and boats \ The pressure exhibits a wide spectrum of applications and is responsible for the flight of aircrafts into air (Figure 1).



ISSN 2229-5518

Figure 1: Some important applications of the Law of Hydrodynamic Pressure. The pressure plays a key role in power plants (A), piping an plumbing system (B) and wind turbines (C).

Studies of hydrodynamic pressure have given rise to Law of Hydrodynamic Pressure, which is equally applicable in fluid dynamics as well as in aerodynamics. In fact, the basic concepts of Aerodynamics are taken from Hydrodynamics e.g. the flow of air over the wings of an aircraft or over the surface of an automobile [2, 3]. The laminar and turbulent flow is also based upon hydrodynamic pressure which is ver important with respect to the shapes of ships and aircrafts (Figure 2). Moreover, hydrodynamic pressure has wide uses in the field of pharmacy and medical such as causes and cure of blood pressure [4]. Whenever we will talk about the air pressure or water pressure we must have to know the concepts of hydrodynamic pressure in detail. This manuscript describes the details of Law of hydrodynamic pressure in a simple way.

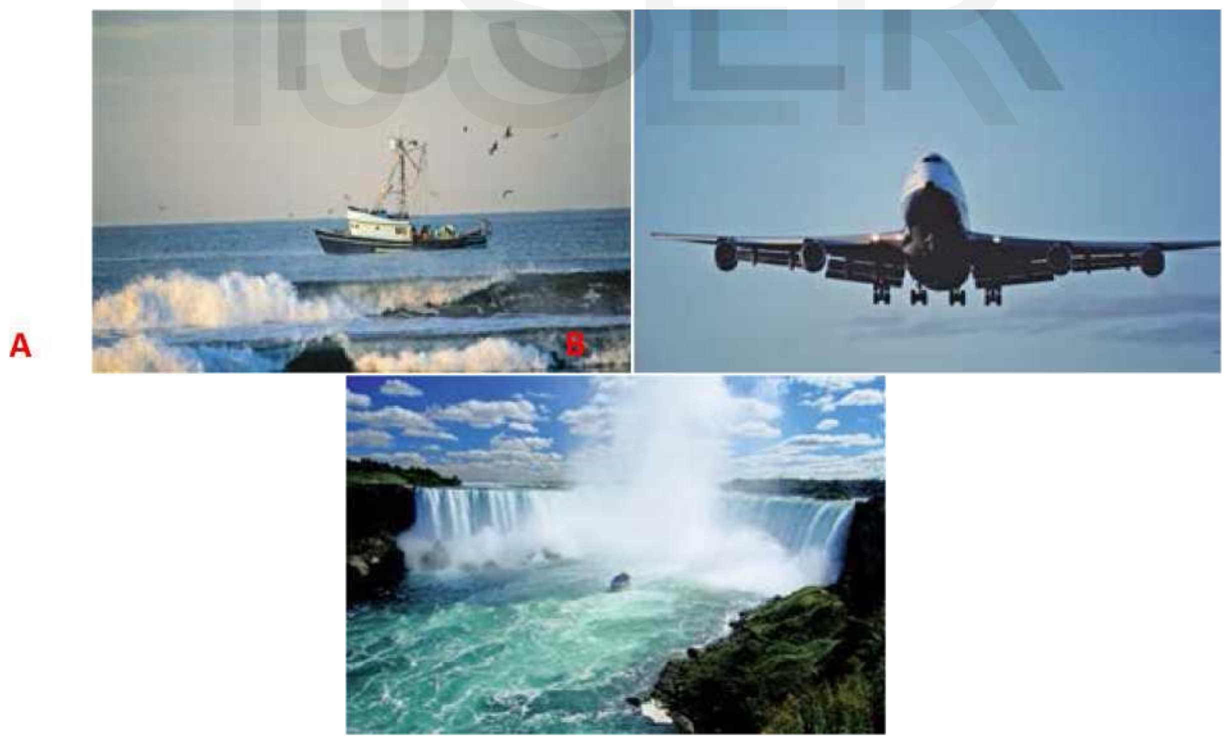


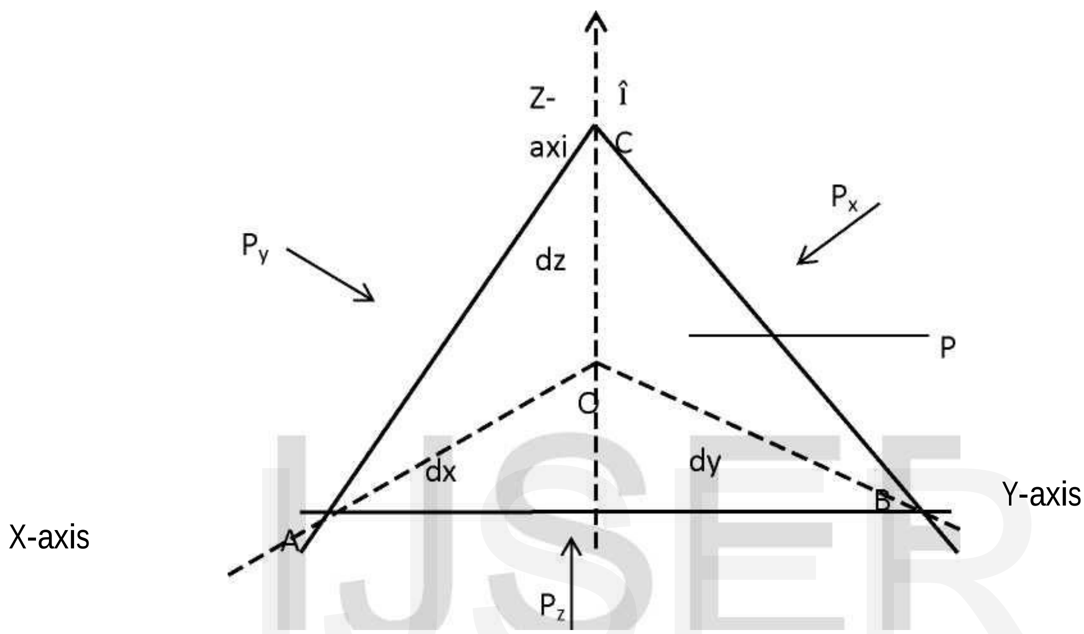
Figure 2: Showing the objects i-e (A) Boats, (B) Aircrafts and (C) Wind blades, which are designed by considering hydrodynamic pressure.

Key words:

Pressure, dynamics, gravitational forces, volume of tetrahedron.

Mathematical form:

Consider the motion of an inviscid fluid in the form of tetrahedron OABC.Let the fluid moving with velocity V and let its density be p.



The Normal vector to side ABC is

$AB\ xAC$ (1)

In triangle OAB

$OA+AB = OB$

$dxi+AB=dyj$

AB=dyj-dxi

In triangle OAC

$OA+AC=OC$

dxi+AC=dzk

Put in equation (1)

$ABxAC$ = (dyj-dxi)x(dzk-dxi)

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \check{k} \\ -dx & dy & 0 \\ -dx & 0 & dz \end{vmatrix} = (dydz)\hat{\imath} - (-dxdz)\hat{\jmath} + (dxdy)\check{k}$$

Let px,py,pz &p be the pressure per unit area on the faces $\overrightarrow{OBC}$,$\overrightarrow{OAC}$,$\overrightarrow{OAB}$ & $\overrightarrow{ABC}$ respectively.

$P=F\backslash A \quad => \quad p_x A_x=F_x$

$p_y=F_y\backslash A_y \quad => \quad F_y=p_yA_y$

$P=F\backslash A \quad => \quad F=pA$

Area of triangle OBC, OAC, OAB & ABC are

$Ax=1\backslash 2dydz$

$Ay=1\backslash 2dxdy$

$A=1\backslash 2(\overrightarrow{AB}\times\overrightarrow{AC})$

$A=1\backslash 2(dxdz\hat{\imath}+dxdz\hat{\jmath}+dxdy\check{k})$

$=> \quad F_x = p_x(1\backslash 2dydz)$

$=> \quad F_y = p_y(1\backslash 2dxdz)$

$=> \quad F_z = p_z(1\backslash 2dxdy)$

$F=p1\backslash 2(dydz\hat{\imath}+dxdy\hat{\jmath}+dxdy\check{k})$

Now

$F=F_x\hat{\imath}+F_y\hat{\jmath} + F_z\check{k}$

$1\backslash 2p(dyd\hat{\imath}+dxdz\hat{\jmath}+dxdy\check{k}) = 1\backslash 2p_x dydz\hat{\imath} + 1\backslash 2p_y dxdz\hat{\jmath} + 1\backslash 2p_z dxdy\check{k}$

$1\backslash 2pdydz\hat{\imath}+1\backslash 2pdxdz\hat{\jmath}+1\backslash 2pdxdy\check{k}-1\backslash 2p_x dydz\hat{\imath} - 1\backslash 2p_y dxdz\hat{\jmath} - 1\backslash 2p_z dxdy\check{k}=0$

$1\backslash 2(p-p_x)dydz\hat{\imath} + 1\backslash 2(p-p_y)dxdz\hat{\jmath} + 1\backslash 2(p-p_z)dxdy\check{k} = 0$

Due to these forces the fluid element may be subjected to the body forces (gravitational forces).

Let $\vec{F}$ be the body forces per unit mass within tetrahedron.

The volume of tetrahedron is:

$V=1\backslash 6\ (dxdydz)$

The total force acting on tetrahedron is $\vec{F}$.m

$\vec{F}.m=1\backslash 2(p-p_x)\ dydz\hat{\imath} + 1\backslash 2(p-p_y)\ dxdz\hat{\jmath} + 1\backslash 2(p-p_z)\ dxdy\check{k} + \vec{F}.m$

$=1\backslash 2(p-p_x)\ dydz\hat{\imath} + 1\backslash 2(p-p_y)\ dxdz\hat{\jmath} + 1\backslash 2(p-p_z)dxdy\check{k} + \vec{F}\rho\ v$

$=1\backslash 2(p-p_x)\ dydz\hat{\imath} + 1\backslash 2(p-p_y)\ dxdz\hat{\jmath} + 1\backslash 2(p-p_z)dxdy\check{k} + \vec{F}\backslash 6\rho dxdydz$

Newton's second law of motion^4

$\vec{F} = \overrightarrow{ma}$

$1\backslash 2(p-p_x)dydz\hat{\imath} + 1\backslash 2(p-p_y)dxdz\hat{\jmath} + 1\backslash 2(p-p_z)dxdy\check{k} + 1\backslash 6\vec{F}\rho dxdydz = \rho v dv\backslash dt$

$1\backslash 2(p-p_x)dydz\hat{\imath} + 1\backslash 2(p-p_y)dxdz\hat{\jmath} + 1\backslash 2(p-p_z)dxdy\check{k} = 1\backslash 6\rho dxdydz\ dv\backslash dt$

$1\backslash 2(p-p_x)dydz\hat{\imath} + 1\backslash 2(p-p_y)dxdz\hat{\jmath} + 1\backslash 2(p-p_z)dxdy\check{k} = 1\backslash 6\rho dxdydz\ dv\backslash dt - 1\backslash 6\vec{F}\rho dxdydz$

$=1\backslash 6\ \rho(dv\backslash dt-\vec{F})\ dxdydz$

Comparing coefficients of "x" components

$1\backslash 2(p\text{-}p_x)\ dydz = 1\backslash 6\rho(dv_x\backslash dt - F_x)$

$As\ dx \to 0$

$1\backslash 2(p\text{-}p_x) = 0$

$p - p_x = 0$

Similarly comparing the coefficients of 'y' & 'z', we get,

$p - p_y = 0$

$p - p_z = 0$

Hence,

$=> p = p_x$

$=> p = p_y$

$=> p = p_z$

Conclusion:

Equations (A), (B) and (C) show that the pressure due to random motion of molecules is equal to the pressure that molecules of the moving body exerts upon the fluid.

$p=p_x$ ________________________ (A)
$p=p_y$ ________________________ (B)
$p=p_z$ _________________________(C)

References:

1-NA Shah, Ideal fluid Dynamics (2007),A-One Publishers Urdu Bazar Lahore.

2-Cengel YA, Cimbala JM, Fluid Mechanics: Fundamentals and applications(3rd edition)2014,Hill MG New York, Chapter 1,8.

3-Frank MW, Fluid Mechanics(4th Edition),Hill MG New York, Chapter 2,Pressure distribution in fluids.

4-Munson B, Young D, Theodore H, Okiishi, Fundamentals Of Fluid Mechanics Fifth Edition 2006,John Wiley & Sons, Chapter 3rd Elementary Fluid Dynamics.

5-Kistemaker, J., "The Spheroidal State of a Waterdrop," Physica, Vol. 29, 1963, pp. 96-104.